AN ANALYTIC MODEL OF POLITICAL ALLEGIANCE AND ITS APPLICATION TO THE CUBAN REVOLUTION

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PURPOSE

The aim of this paper will be to develop a simple model whose successive solutions will reveal changes in the political environment. The model will be simple enough to be solved analytically and yet complex enough, hopefully, to include the major factors underlying political change. Subsequently, an application will be made to Cuba during the period of political upheaval which resulted in the overthrow of Batista's régime.

The chief difficulty in constructing a simple analytic model is in determining which of the many possible formulations departs least appreciably from reality. In any abstraction crucial variables may be left out or crucial relationships badly stated. Finally, even though the model appears to provide a logical representation of the real system, it may not be able to reproduce accurately its behavior.

THE VARIABLES

The elements of the model will be the participants in political affairs and the "interest" or "pressure" groups to which they belong and through whose collective agency they act. Competition between groups will take the form of proselytizing, the number of adherents that the group draws being the measure of its success. A distinction will be made between "normal" periods and those of great stress, or "crises."

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THE MODEL

The system will contain N individuals, belonging to one or the other of two "interest" groups, I and II. The number of individuals belonging to Group I will be designated as N_I and the number belonging to Group II as N_{II}:

$$N = N_I + N_{II} \quad . \qquad \text{Equation (1)}$$

Individuals will be categorized according to characteristics which would lead them to behave differently; characteristics which might give rise to different behavior could be age, sex, wealth, or more complex and multi-dimensional attributes such as social status or political ideology. The particular characteristics that we shall use are the degree of commitment of the individual to the group and the extent of the contribution that he makes to it. Individuals will be assumed to fall into one of three categories: those who have joined the group, but who give it no more than their sympathy; those who have joined the group and are active in securing its aims; and finally, those whose fortunes are so closely interwoven with those of the group that their adherence is irrevocable. In Table 1, these three different types of individuals are called loyalists (L_I and L_{II}), activists (A_I and A_{II}), and the committed (C_I and C_{II}) respectively. The total number of members of Group I (N_I) is thus equal to those who are loyal to it(L_I), those who are actively engaged in its affairs (A_I) and those who are irrevocably committed to it (C_I):

$$N_I = L_I + A_I + C_I \quad . \qquad \text{Equation (2)}$$

Similarly for Group II, the total number of members is equal to

$$N_{II} = L_{II} + A_{II} + C_{II} \quad . \qquad \text{Equation (3)}$$

Our reasons for dividing individuals into three categories -- loyalists, activists, and committed -- are twofold. First, we wish

Table 1

DISTRIBUTION OF MEMBERS AMONG TWO INTEREST GROUPS

Group I		Group II		Total	
Category	Number	Category	Number	Category	Number
Loyalists	L_{I}	Loyalists	L_{II}	Loyalists	L
Activists	A_{I}	Activists	A_{II}	Activists	A
Committed	C_{I}	Committed	C_{II}	Committed	C
Members	N_{I}		N_{II}		N

to make a distinction between those individuals who are capable of changing their allegiance, and those who are not. We shall assume that individuals who are loyal to one group can shift their loyalty to the other, and similarly that those who work actively for one group can apostatize and become equally active for its competitor. But those who are already committed are committed permanently. The design of the model thus will permit loyalists to change their affiliation and activists to change theirs, but it will not permit a change from one category to another. Loyalists will remain loyalists, and activists activists, regardless of their allegiance.*

As the groups compete one with another for members, the numbers of individuals affiliated with each will change. In trying to understand the changes, we shall focus on two sets of phenomena: the distribution of individuals amongst the two groups at different instants in time (i.e., obtaining numbers for L_I, A_I, L_{II} and A_{II} at t_1, t_2, etc.) and the rates of change of allegiance with respect to certain variables (e.g., $\frac{\partial N_I}{\partial A}$, $\frac{\partial N_{II}}{\partial A}$, etc.) at the point of solution. The distribution of members among the two interest groups can only be determined when there is a balance of power between the two groups; balance of power being equivalent, in terms of the physical system of which our model is the analogue, to the existence of equilibrium (where individuals neither enter nor leave the system and where those contained within it may shift from one group to another, but never without

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*We do not argue that individuals are inherently stable, while simultaneously being politically variable. Shifts from one category to another are conceivable; for example, an individual who has been loyal to Group I in the past but who has contributed to it no more than his presence may become active on its behalf, thus shifting from the category of loyalists to that of activists. In practice, individuals may move from any of the two categories of loyalists and activists (in either Group I or II) to any of the other categories.

But we do have to assume that all movement takes place within categories in order to keep the model simple, for allowing movement from one category to another -- activist to loyalist or _vice versa_ -- complicates the system considerably and precludes analytic solution. We shall, however, keep this assumption in mind and try to indicate subsequently what changes might occur if it were relaxed.

compensating movement in the opposite direction). In equilibrium, there is, within the system itself, no potential for change; if left alone, the proportions in each group of the different categories of individuals and of all individuals as a whole will remain constant. Uncompensated-for movements will occur only if entry or departure occur or if the fundamental nature of the system is altered.

Shifts in the "balance of power" are equivalent to a "transient state," i.e., to the period of adjustment to a (perhaps new) equilibrium. If some disturbance upset the balance between the two groups, there would be changes of allegiance from that group which was now relatively less attractive to that group which was more; were it possible to obtain solutions at different instances during the period of adjustment, they would indicate changing numbers of individuals in the different groups. But this we will not be able to do, for only at equilibria are the distributions calculable.

EQUILIBRIUM DISTRIBUTIONS

There are two methods by which we could determine the distribution of members of the two interest groups in equilibrium. One would be the forthright one of observing the system and counting the number of individuals who belong in each of the categories. If the observation were carried out during a period in which there were no uncompensated movements in any direction, then this would yield a representation of the system in equilibrium. Subsequently, using these empirical data, we could derive the general characteristics of the system.

The other method of estimating the distribution of members of the two groups -- differing not in theory but in mode of calculation -- would be to determine the underlying characteristics of the system, i.e., its "nature," and then deduce the way in which a given number of individuals would distribute themselves. In applying our model, we shall use both of these methods, first inducing the general characteristics of the Cuban political system from particular data and second deducing the distributions of members among the groups at subsequent dates.

Let us discuss in greater detail the "nature" of the system, in the terms of the physical world from which our model is drawn and also in terms of the political environment that it attempts to describe.* The physical system that we shall choose is the chemical solution, made up of different materials dissolved in one another, and the characteristic on which we shall focus is the total free energy. Utilizing the First Law of Thermodynamics -- that it is impossible to create energy -- and the Second -- that it is impossible to utilize the heat energy in our surroundings continuously to do work: i.e., that the passage of heat from one body to another body at a higher temperature never takes place spontaneously -- it can be determined that the equilibrium state of a system is determined by the condition that the total free energy be a minimum.** Thus in a stable system there is no free energy available; only an unstable system has excess free energy and is capable of doing work. The struggle for existence might be typified as the attempt to exploit free energy.

If G (honoring J. Willard Gibbs) is the total free energy of the system, then in equilibrium $dG = 0$.*** Changes in the total free energy of a system (dG) can come about through changes in the internal energy of its components (dF), the temperature (dT), or the pressure (dP):

$$dG = dF - SdT + VdP \,, \qquad \text{Equation (4)}$$

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*The author is summarizing what he learned from J. C. Dehaven, L. Shapley and N. Z. Shapiro, who kindly and clearly developed the chemical thermodynamics. If the exposition is still murky, the fault is solely his.

**See N. Z. Shapiro and L. S. Shapley "Mass Action Laws and the Gibbs Free Energy Function," Santa Monica: The RAND Corporation, RM-3935-1-PR, September 1964.

***From the point of view of applicability to demographic systems, the steady state (in which entry and exit occur, provided it be at equal rates) is probably a better concept than equilibrium. In the steady state, however, the total free energy of the system (now including entry and exit) is not minimized, i.e., $dG \neq 0$. But in cases where the rates of entry and exit are low relative to the rates of internal reaction, the parameters appropriate for <u>equilibrium</u> systems can be used to determine the configuration at <u>steady state</u> without the errors being significant.

where for the additional items S is the entropy and V the volume. At constant pressure and temperature the second and third terms disappear, leaving changes in the total free energy of the system equal to changes in the internal energy of the components.

With varying temperatures, the second term enters into the equation, usually in a paradoxical fashion. Since F and S are related through the equation

$$S = k \log N (F), \qquad \text{Equation (5)}$$

where k is Boltzmann's constant and N is the number of ways in which the system may contain the energy (F), a variation in the state of the system that decreases F usually increases -TS. There is thus a competition between the first two terms on the right-hand side of equation (4). Since minimizing the entropy term (-TS) requires that the total energy be large whereas minimizing the energy term (F) requires the total energy to be small, the condition that the total free energy be made a minimum will involve some delicate balance. Often the way in which the total free energy is minimized for systems containing a high internal energy (e.g., a system at a high temperature) is through a disorder in the arrangement of the components; examples of disorder being oscillation of atoms, changes in the lattice structure of crystals, or changes of state.

But in applying the theory of chemical thermodynamics to political phenomena we shall consider only changes in the internal energy of the components themselves, or in terms of equation (4), the simplified form

$$dG = dF \quad . \qquad \text{Equation (6)}$$

In equilibrium $dF = 0$, or F, the internal energy, is minimized.

The internal energy of the components, F, is weighted by their concentrations. Assuming that the energy of every component within the

same category is identical, the internal energy of the system is equal to

$$F = \sum_{j=1}^{C} \sum_{i=1}^{N_j} n_{ij} f_j \quad . \qquad \text{Equation (7)}$$

where F is the free energy of the system as a whole, n_{ij} is the i th component in the j th category, f_j is the internal energy of that component, and C and N are the total number of categories and components respectively.

The internal energy of any component in a given category, f_j, is in turn equal to the sum of two items -- first, the standard free energy parameter (f_j^0),* and a second item involving a universal constant (R), the absolute temperature (T) and its activity (a_j):

$$f_j = f_j^0 + RT \ln a_j \quad . \qquad \text{Equation (8)}$$

In very dilute solutions containing no incompatible elements, the value of the activity coefficient a_j approaches that of the concentration of the component.** In this event, the last item in equation (8) can be replaced by the proportion of all of the components of a group (g) which are concentrated within a single given category (k): i.e., $\dfrac{\sum_{i=1}^{N_k} n_i}{\sum_{j=1}^{C_g} \sum_{i=1}^{N_j} n_{ij}}$.

*Also known as the Gibbs free energy parameter; both are unfortunate terms, annoyingly similar to F (the internal energy) and G (the total free energy).

**See G. B. Dantzig, et. al., "A Mathematical Model of the Human External Respiratory System," Perspectives in Biology and Medicine, Vol. IV, No. 3 (Spring 1961), pp. 339-342.

In this case

$$f_j = f_j^0 + RT \ln \frac{\sum_{i=1}^{N_k} n_i}{\sum_{j=1}^{C_g} \sum_{i=1}^{N_j} n_{ij}} . \qquad \text{Equation (9)}$$

For example, in the system described in Table 1, the concentration of activists in Group I is equal to $\frac{A_I}{N_I}$.

This is as far as we shall go in the derivation of the theorems governing the chemical systems. Our aim subsequently will be to attach physical significance to the terms. The standard free energy parameter, f_j^0 in the equations (8) and (9), is a fundamental characteristic of the components belonging to the category, akin to their color or their density. It is elemental, inherent, basic, unaltering. A molecule of water in a liquid state has a standard free energy parameter that is invariant and particular to itself, because of the atoms of which it is composed, their configuration, the state in which it is existing, and all the other things that are unique to it. A molecule of water vapor also has its own standard free energy parameter, different from that for liquid water because it is more agitated, which enables it to exist in a more elevated state.

The last term in equation (8) and equation (9), $RT \ln \frac{\sum_{i=1}^{N_k} n_i}{\sum_{j=1}^{C_g} \sum_{i=1}^{N_j} n_{ij}}$

reflects the fact that the attributes of a component change with changes in its environment. If the temperature rises or if the concentration grows, its free energy increases. But, unlike the standard free energy parameter the rise in temperature or the increase in concentration are

outside factors, beyond the component's molecular boundaries. To take account of both factors, internal and external, we combine the two terms: the resultant is the internal energy.

In our analogue, combining the two terms in equation (8) or (9) is equivalent to considering both personal and social factors in determining an individual's political complexion. The resultant could be thought of as measuring his affinity for the particular group considered: the greater his affinity, the less his agitation or his "free energy." Equilibrium in the political system would be attained by the pledging of allegiance on the part of each of the individuals to the particular group for which he had the greater affinity. Only when all individuals, taken collectively, were least "agitated" (in chemical terms when the total free energy were minimized) would their distribution amongst the groups be stable.

EQUILIBRIUM COEFFICIENTS

As we stated earlier, the standard free energy, f_j^0, of a component is something that is innate to it, one of its elemental characteristics. The standard free energy parameter is calculated from the specific heats of its ingredients and the heat of the reaction by which it is formed. But as is the case with most elemental phenomena, the actual values of the standard free energies are known for very few substances; the more common way of estimating the standard free energy parameter is to determine it through empirical observation.

The intermediary in this mode of estimation is the equilibrium coefficient, designated K usually.* The equilibrium coefficient measures the proportions in which various chemical components are present at equilibrium. Many chemical reactions are reversible, meaning that the proportions in which the two sets of components -- reactants and products -- exist in equilibrium will be identical no matter what were the initial proportions. The equilibrium coefficients are calculated by placing all of the elements on one side of the

*Ibid.

reaction in the numerator and all of the elements on the other side of the reaction in the denominator. If we imagined loyalists, L, in group II moving to group I (or, since the reaction would be reversible, loyalists in group I moving to group II), then the equilibrium coefficient for loyalists, K_L, would be equal to

$$K_L = \frac{\dfrac{L_{II}}{N_{II}}}{\dfrac{L_I}{N_I}} , \qquad \text{Equation (10)}$$

similarly, for activists, K_A would be equal to

$$K_A = \frac{\dfrac{A_{II}}{N_{II}}}{\dfrac{A_I}{N_I}} . \qquad \text{Equation (11)}$$

Two additional equilibrium coefficients for the committed, together with K_L and K_A, would specify completely the distribution of individuals among the two groups of Table 1.*

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*Two are required because those who are committed to Group I cannot exist in Group II and *vice versa*: there are really not one, but two different, immobile species, $C_{(I)}$ in groups I and II, and $C_{(II)}$ in groups I and II.

The equilibrium coefficients for the committed will be

$$K_{C_{(I)}} = \frac{\dfrac{(I)_{II}}{N_{II}}}{\dfrac{C_{(I)_I}}{N_I}} , \qquad \text{Equation (11a)}$$

and

$$K_{C_{(II)}} = \frac{\dfrac{C_{(II)_{II}}}{N_{II}}}{\dfrac{C_{(II)_I}}{N_I}} . \qquad \text{Equation (11b)}$$

Note that the coefficient which specifies the distribution of specific components in the system at equilibrium contains not only terms for the components themselves (L and A in K_L and K_A respectively) but also for all the other components in each group. The equilibrium coefficient K_L is thus equal to the ratio of the concentration of L in group I to that in group II; similarly, the equilibrium coefficient K_A is equal to the relative concentration of A in each of the two groups. An equilibrium coefficient equal to 1.00 would signify equal concentrations of the element in each of the two groups -- equal concentrations, not equal numbers. For example, if K_L were equal to 1.00 and the fraction of group I comprised of loyalists were equal to 0.50 -- in other words, one out of every two members of group I were a loyalist -- then the fraction of the loyalists in group II would also be 0.50.

From this we can see that the value of the equilibrium coefficient can assume any number from zero to plus infinity, depending on the "affinity" or "nature" of the element whose equilibrium coefficient is being calculated. In other words, K_L can vary between zero and plus infinity, depending on the "nature" of L; and K_A upon the "nature" of A. For the case which we have taken, where L and A do not interact, the respective K's and consequently the distributions amongst the two groups at equilibrium are dependent upon the "nature" of the component to which they relate and upon the number of other components in the group. If K_L were very large, it would mean that the concentration of L in group II would be relatively higher than that in group I; i.e., that for L the gradient from group II to group I would be very steep. If K_L were less than unity, the consequence would be that the concentration of L in group I would be higher than that in group II. And the "cause" of the relative concentration's being higher in one group or the other would be that the component in question had a greater affinity for the one or the other.

We evaluated the equilibrium coefficients from the empirical data on the relative concentrations at equilibrium. Now we shall show how the same equilibrium coefficients could have been derived from a know-

ledge of the inherent nature of the ingredients. The relationship is very simple: at equilibrium the K's are related to the standard free energy parameters as follows:

$$\Delta f_j^0 = -RT \ln K_j \quad , \qquad \text{Equation (12)}$$

where R and T are the universal constant and the absolute temperature, as before. The Δf_j^0 indicates that what is being calculated is not the standard free energy parameter of components of category j in one group but rather the difference between the standard free energy of components of category j in group II and that of the same components in group I. Using the items of Table 1, we could, for example, apply equation (12) to activists, writing:

$$\Delta f_A^0 = -RT \ln K_A \quad . \qquad \text{Equation (12a)}$$

From equation (12) or (12a) it can be seen that the higher is the temperature, the higher is the standard free energy and the greater the ease with which a reaction will occur. Conversely, the higher is the standard free energy, the higher is the value of K and the further the reaction will proceed.

A GENERAL SOLUTION

With values either for the standard free energy parameters or for the equilibrium coefficients, one can determine the equilibrium conditions for the system by minimizing the total free energy, subject to mass balance constraints; i.e., subject to the requirement that elements neither be created nor destroyed. Assuming that it is the values of the equilibrium coefficients that have been obtained, the solution for the model which is outlined in Table 1 is

$$1 = \frac{L}{S + K_L (N - S)} + \frac{A}{S + K_A (N - S)} + \frac{C_I}{S} \quad , \qquad \text{Equation (13)}$$

where L is the total number of loyalists; A the total number of

activists; C_I is the number committed to group I; K_L and K_A are the equilibrium coefficients; and S is a dummy variable. Of the three roots of S, that one which is a real number between 0 and N will be equal to N_I. If there is no such root, it indicates that the system is degenerate -- i.e., that the nature of the elements and of their interaction is such that one of the two groups cannot exist in the steady state under all possible types of stress.*

This is the way a chemical system is analyzed; the analysis of a political system could be carried out in the same manner, using either of the two methods: observing the political environment in a steady state, (i.e., observing all the values L_I, L_{II}, A_I, A_{II}, C_I, C_{II}) and then inducing the equilibrium coefficients; or estimating the free energies (i.e., the affinities of different individuals for the different groups, or, looking at it from the other point of view, the pressure that would have to be put upon them in order to change their allegiance) and via the equilibrium coefficients deducing the equilibrium solution. Using the first method, one would calculate the equilibrium coefficients K_L and K_A, as in equations (10) and (11). Using the second method, one would determine the equilibrium conditions by minimizing the total free energy for the system as a whole (equation (7). The qualification that we would have to make in applying the general solution of equation (13) would be that it describes a very simple political system in which there exist only two groups and three categories of individuals and in which movement from one group to another is permitted only for individuals within the same category. Whereas we can obtain the general equilibrium solution for the two-group, three-category system, for any more complex we could obtain only particular solutions and could not be certain that we could acquire enough particular solutions so as to be able to determine the behavior of the system over the entire range of possible outcomes. Hopefully, the

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*J. C. DeHaven and N. Z. Shapiro, "Intrinsic Control of Body Fluid...," Santa Monica: The RAND Corporation, RM-4609-PR, July 1965, pp. 6-8 and N. Z. Shapiro, "Primitive Chemical Equilibrium Systems," Santa Monica: The RAND Corporation, RM-4464-PR (in preparation).

example which we shall introduce later is not so much more complex than this.

DEPARTURES FROM EQUILIBRIUM

The methods described above enable us to determine, either from empirical observations or from the standard free energy parameters of the constituents, what the distributions would be at any other equilibrium poi-' For example, once we had observed an initial equilibrium or obtained the values of the standard free energies, we could determine the resulting distribution of members among the two groups following changes in the total number of loyalists, of activists, of committed, or of any combination of these.

But we might also be interested in what would happen at the margin -- were a very few individuals to be added to or subtracted from the system or were the equilibrium coefficients to alter slightly. In cases where the increments, be they entrants or departures or changes in the values of the K's, are extremely small relative to the parameters of the system, we would find that the Jacobian matrix provided a convenient way of determining the consequences.* The Jacobian matrix is a table of partial derivatives; each partial derivative measures the change in the number of individuals in one category in one group relative to changes in some overall category: for example $\frac{\partial A_I}{\partial A}$ would be the increase of activists in group I resulting from an increase in activists in the system as a whole and $\frac{\partial A_I}{\partial K_A}$ the increase resulting from an increase in the value of the equilibrium coefficient. There will be as many partial derivatives in each set -- entrants or

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*See N. Z. Shapiro, et. al., "The Variation of the Parameters of Chemical Equilibrium Systems," Santa Monica: The RAND Corporation, RM-4465-PR (in preparation).

equilibrium coefficients -- as there are [categories]^2 [groups], or in the case of the system described in Table I, eighteen.*

In theory, the values of the partial derivatives can be either negative or positive; they can even be greater than unity, implying, for the partial derivative relative to entrants that adding an individual of one category to the system would increase the number of individuals in that category in <u>one</u> of the groups by more than one, while actually reducing the number of individuals in that category in the other group. In this case adding an individual to the political system would not only result in that individual's joining a particular group, but would also, through his adherence, exert an irresistible attraction upon someone already committed to another group. In these cases, the system is extremely sensitive to the addition or subtraction of a small number of elements.

Although the standard free energies and the equilibrium coefficients may remain constant while the numbers of individuals of different types are changing, or the numbers of individuals constant while the equilibrium coefficients are changing, the partial derivatives will not. Each different equilibrium will have a different Jacobian. It is conceivable, and in chemical systems often occurs, that the values for the partial derivatives can change their sign (from plus to minus or from minus to plus) as progressively greater amounts of one element are added or larger changes in one coefficient observed. In the former case, this is equivalent to saying that as progressively greater numbers of one type of individual enter the system, their total membership in one of the two groups could either rise to a maximum and fall again, or fall to a minimum and then rise (or what is similar, it could rise or fall to the point where the system has degenerated and all of the individuals adhere to one group.)

**There will also be a number of partial derivatives, equal to the number of categories multiplied by the number of groups or six in our case, which are the sum of the individual partial derivatives of the different categories within each group. For example, $\frac{\partial N}{\partial L}$ would be equal to $\frac{\partial L_I}{\partial L} + \frac{\partial A_I}{\partial L} + \frac{\partial C_I}{\partial L}$. And, of course, $\frac{\partial N_I}{\partial L} + \frac{\partial N_{II}}{\partial L}$ would be equal to unity.

AN HISTORICAL EXAMPLE

We shall now illustrate our simple two-group, three-component model by applying it to the Cuban revolution of 1956-1958. Our procedure will be to observe the distribution of politically conscious individuals among the competing groups at some instant when the political situation in Cuba was stable, and with these data calculate the equilibrium coefficients. Using the equilibrium coefficients, we shall then estimate what was the distribution of Cubans among the groups at successive dates beyond the initial equilibrium, in an effort to determine if the results of the model bear any similarity to those that actually occurred. If they do, then by analyzing the "nature" of the system itself and of the alterations that were made to it we may be able to understand better the political changes that took place.

We shall assume that the political forces in Cuba were "balanced" in 1953. It was during this year that Castro's coup failed, for lack of any general support, and that Batista was preparing to legalize his seizure of power by seeking election to the presidency of the Cuban republic. In this year the political system appeared stable, with Batista firmly in control but with his opponents maintaining their modest strength.

Of the Cuban population of 5,829,000, only approximately half a million were allied politically with Batista's régime or the opposition, according to the data in Table 2.* "Loyalists" signifies those who were in sympathy with a group but who lent little more than their presence to it; "activists," those who were engaged deliberately, energetically,

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*The source of the population statistic and other demographic data is Groupo Cubano de Investigaciones Economicas, Un Estudio sobre Cuba, Miami, University of Miami Press, 1963. The prime and invaluable sources of the estimates of allegiance were Boris Kozolchyk and Ralph Sarre of The RAND Corporation, recalling Cuba as it was in this period. They identified the classes from which each group drew its supporters: in the cases where the class was numerous, they estimated the percentages that fell in each category; in the cases where the class was small, they estimated the numbers directly. Both the individual figures and the totals are therefore subjective.

Table 2

OBSERVED DISTRIBUTION OF FOLLOWERS AND OPPONENTS OF THE CUBAN RÉGIME, 1953*

Category of Members	Opponents	Followers of Batista's Régime	Total
Loyalists	50,000	400,000	450,000
Activists	10,000	50,000	60,000
Committed	5,000	50,000	55,000
Total	65,000	500,000	565,000

Note:

[a]The relative numbers of individuals in each of the categories in 1953 is very important, for they will determine the distribution of adherents at all subsequent equilibria. Starting with those who were loyal to the opponents of Batista's régime, approximately half (27,000) of the 50,000 total were students in the upper years of the high schools and in the university; this represents a little more than half of the adolescent and adult male students. Aversion to the régime was chronic at the University of Havana, which then had a student body of approximately 25,000. Three other occupational classes -- businessmen (2,000), urban laborers (10,000), and professionals (1,000) -- and one political -- roughly 7,000 Communists -- provided nearly all of the remainder of the loyalists. (The different classes are all mutually exclusive.) Finally, there were a few among the landowning (200) and other classes (2,800) antagonistic to the existing government.

The Communists contributed the largest number of activists to the opposition, perhaps half (5,000) of the total of 10,000. Substantial numbers of activists also came from those classes which contributed to the category of loyalists -- students (1,000), businessmen (500), laborers (1,000), the professions (1,000), and others (1,500). The backgrounds of those who were unshakeable in their opposition to Batista were much the same -- Communists (3,000), laborers (1,000), businessmen (500), students (500) and others (1,000).

Batista drew his support from all of the elements in the Cuban society. The largest single classes, accounting for three-quarters of the total loyal to Batista's régime, were probably the government bureaucracy (100,000), labor (100,000), and women (100,000). Additional supporters were drawn from business (20,000), wealthy landowners (500), older students (9,000), the armed forces (7,000), the *guardia rural* (1,000), the professions (500), and other classes (62,000).

Those active in the support of Batista's régime were drawn primarily from the bureaucracy (25,000), and to a lesser extent, from business (10,000). The labor unions provided many active supporters (5,000), and the student body (1,000), landowners (400), armed forces (2,000) *guardia rural* (1,000), the professions (500), and other classes (5,000)

<u>Note to Table 2</u>

lesser numbers. Amongst those who were irrevocably committed to Batista's régime, the bureaucracy accounted for over half (30,000) and the business community one-fifth (10,000). The remainder were distributed among professionals (500), labor (1,000), the armed forces (1,000), <u>guardia</u> <u>rural</u> (1,000), wealthy landowners (100), and others (7,400).

and often violently in furthering their group's endeavors; and the "committed," those whose fortunes were so tied up with one or the other group as to have been politically immobile. In 1953, the existing régime commanded far more support in each of the three categories than did its opponents, the number of loyalists being eight times that in the opposition; the number of activists, five times; and the number of committed, ten times.

Using the data on the allegiance of Cubans in 1953, we shall calculate the equilibrium coefficients, and, assuming them to have remained constant -- i.e., assuming that the relative attractiveness of the two groups did not change -- determine subsequent equilibria as the number of participants increased. The equilibrium coefficients for loyalists, defined as the concentration of loyalists in Batista's régime relative to that in his opponent's, equals 1.04; while the equilibrium coefficient for the activists equals 0.65. From our discussion of the significance of the equilibrium coefficients, we recognize that those whose nature would lead them to be loyal to a particular régime, but not particularly active in its affairs, would have had an affinity for Batista's régime, whereas those who entered actively into political events would have tended to join the opposition. If those entering (leaving) politics had a predeliction for action, the opposition (régime) would have benefited; if they tended to be loyal but docile, the results would have been the reverse; if those who entered or left the arena were some of one and the rest of the other proclivity then the changes might have benefited either group, depending upon the relative proportion in which individuals on the two categories were added or subtracted and the overall distribution that had already been achieved.

But what we wish to do is to see if our model will reproduce with some accuracy the changes that actually took place in Cuba. Between 1953 and 1956, when Fidel Castro returned to Cuba and initiated his rebellion in the Sierra Maestre, the stresses in the society grew. The numbers of individuals actively engaged in politics did not change appreciably, but the increasing harshness of the struggle led many of

those who had been passive to withdraw their allegiance. Therefore although the total numbers of activists would have been approximately the same as in 1953, by 1956 the total number of loyalists would have been reduced by about 100,000. (There was believed to have been no change in the number of those who were irrevocably committed to each group.)

What would have been the distribution of the politically conscious among the two groups in 1956, according to our model? Assuming that the "nature of the political system did not change, and that in 1956, as in 1953, it could have been represented as being at equilibrium, the new distribution would have been that of Table 3. Of the total of 350,000 loyalists in 1956, 42,000 would have been opposed to Batista and 308,000 sympathetic to him. Of the 60,000 activists, 11,000 would have agitated for the opposition and 49,000 for Batista.

It is important to understand that we have utilized the estimates of the members of politically conscious only to the extent of concluding that the total member of loyalists diminished by 100,000 between 1953 and 1956, and that the total numbers of activists and the committed remained unchanged. We did not, as we did in 1953, try to observe their distribution among the groups: these were obtained through the mechanism of the model. The "nature" of the system incorporating the data of 1956 is identical to the "nature" of the system incorporating those of 1953: i.e., the "free energies," or the relative attractiveness of the different groupings, is unchanged.

But the proportions in which the loyalists and the activists fell into the two groups has changed. Of the 100,000 loyalists who withdrew their allegiance, in responding to the pressures of the system 92,000 would have deserted the existing régime and only 8,000 the opposition. The "balance" would have swung slightly in favor of the opposition, for its share of the total number of loyalists would have increased from 11 to 12 per cent.

Although the total number of activists in the system did not change between 1953 and 1956, according to the model their relative distribution at equilibrium among the two groups would have -- those in opposition increasing from 10,000 to 11,000; and those supporting the régime

Table 3

CALCULATED[a] DISTRIBUTION OF FOLLOWERS AND OPPONENTS OF THE CUBAN RÉGIME, 1956

Category of Members	Opponents	Followers of Batista's Regime	Total
Loyalists	42,000 (Calculated)	308,000 (Calculated)	350,000
Activists	11,000 (Calculated)	49,000 (Calculated)	60,000
Committed	5,000	50,000	55,000
Total	58,000 (Calculated)	407,000 (Calculated)	465,000

Note:

[a]For these and subsequent numerical results, the author is indebted to Mrs. M. Shapley, who applied to the Cuban political system the computer program that Harold Cantor, R. J. Clasen and she were instrumental in developing for the calculation of chemical equilibria.

diminishing from 50,000 to 49,000. The proportion in opposition would thereby have increased from 17 to 18 per cent.

If the total number of activists did not change, why should their distribution? The reason is that the departure of loyalists upset the balance between the two categories. Since loyalists had a predeliction for Batista's régime, it, rather than the opposition, would have tended to lose more by their departure. But their apostasy would have left too large a proportion of all the activists supporting Batista and too small opposing him: too large and too small according to the affinities of the categories of individuals for the two groups and for one another. As a consequence, to restore the "balance" a thousand activists would have had to have transferred their allegiance from Batista to his antagonists.

The increase in the relative strength of Batista's opponents between 1953 and 1956 demonstrated by the model reproduces accurately the actual increase that was believed to have occurred. Those observers of events in Cuba at this time upon whose experience we have drawn concurred that it was the existing régime which suffered a greater loss of adherents in the category of loyalists, and that a small portion of those who had formerly been active in its behalf shifted to the opposition. As to the numerical strength of each group in 1956, the calculated distribution appears to be substantially correct, which is about as precise a statement as we can make with subjective data.

"TRENDS" EXISTANT IN 1953

That at this stage in the development of political affairs in Cuba the opponents of the régime were bound to have benefited from a reduction in the total number of loyalists can be seen by observing their partial derivatives (see Table 4). In 1953, for each hundred Cubans who ceased being politically loyal (the other categories remaining fixed) seven would have deserted the opposition and ninety-three Batista.*

*The partial derivative representing the decrease in the number of loyalists in opposition for each unit decrease in the total number of loyalists appears in the second row, first column of Table 4; that for the decrease in the total number of loyalists in Batista's régime for each unit decrease in the total number of loyalists in the sixth row, first column.

Table 4

JACOBIAN MATRICES FOR MARGINAL CHANGES IN THE DISTRIBUTION OF MEMBERS, RELATIVE TO CHANGES IN THE TOTAL NUMBERS OF POLITICALLY CONSCIOUS, 1953

Category of Members / Group and Category of Members	Loyalists	Activists	Committed
Opponents	0.07	0.74	10.8
Loyalists	0.07	0.48	8.3
Activists	-0.01	0.26	1.6
Committed	0.00	0.00	1.00
Followers of Batista's Régime	0.93	0.26	2.28
Loyalists	0.93	-0.48	1.08
Activists	0.01	0.74	0.20
Committed	0.00	0.00	1.00

That the desertion of loyalists would have precipitated a shift of <u>activists</u> from support of Batista to that of his opponents can be seen from the sign of the appropriate partial derivative, which is negative, indicating that a departure of loyalists from the system would increase slightly the number of activists in opposition.*

By looking at Table 5 we can see the consequences of, first, changing the number of loyalists while holding constant the number of activists; and, second, changing the number of activists while holding constant the number of loyalists. Adding loyalists or subtracting activists would have benefited Batista; subtracting loyalists or adding activists would have benefited the opposition. To illustrate the persistence of the trends, let us follow through with the last of these alternatives, namely adding successively more activists to the system. The data are given in the bottom half of Table 5, where the total number of loyalists in the system is held constant at 450,000 (column one) and

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*Making use of the partial derivatives we could calculate the ratio of departing loyalists to departing activists which would have maintained the <u>status quo</u> (defining <u>status quo</u> arbitrarily as no change in the relative membership of the groups: i.e., $\frac{N_I}{N_{II}} = \text{constant}$).

Let y be the fraction of departing loyalists and (1 - y) the fraction of departing activists. Then, to preserve the <u>status quo</u>,

$$\frac{N_I}{N_{II}} = \frac{y \frac{\partial N_I}{\partial L} + (1 - y) \frac{\partial N_I}{\partial A}}{y \frac{\partial N_{II}}{\partial L} + (1 - y) \frac{\partial N_{II}}{\partial A}} . \qquad \text{Equation (21)}$$

In 1953, the ratio of the total membership of the two groups was $\frac{65,000}{500,000}$ or 0.13. To have maintained this ratio constant, the fraction of apostates who were passive by nature (y) would have had to have been 0.93. The implication is that to have maintained the <u>status quo</u>, e.g., to have compensated for the departure of each ninety-three loyalists, the existing regime would have had to have forced out of the political arena seven activists. It takes only a few who are militant to offset many who are mild.

the number of activists permitted to increase by increments of 60,000 from 60,000 to 360,000 (column three). At the beginning (in 1953, row seven), of the total of 60,000 activists, only 10,000 were in opposition. Doubling the total number of activists would have more than trebled the number of activists in opposition (row eight, column four) and nearly doubled the number of loyalists in opposition (row eight, column two), the additional activists having "pulled" loyalists into an opposition as well. If successively more individuals had become politically active, the size of opposition would have grown to the point where, between 180,000 and 240,000 activists (rows nine and ten), the overall balance would have shifted: for with 180,000 activists in the system, the opponents to the régime would have won the allegiance of 46 per cent; with 240,000 they would have attracted 59 per cent. If to our measure of political power we also added those who were committed to the régime or to the opposition, we would have found that the shift in the balance took place when there were between 240,000 and 300,000 activists in the system, for at this point the number of activists plus the number of committed in opposition would have exceeded the numbers of Batista's forces. Using the total membership -- loyalists, activists, and committed -- as the measure, we would have found the shift in the balance of power to have occurred in the same interval.

The total response of the system to all possible changes in the number of participants can be visualized from Fig. 1. There, changes in the fraction of activists in opposition are plotted against both changing numbers of loyalists (drawing on the data in the upper half of Table 5) and changing numbers of activists (the data in the lower half of Table 5). All points with coordinates equal to the fraction of activists in opposition lie on the surface which rises monotonically to a peak at the far right-hand corner of the figure. The surface displays the characteristic mentioned earlier -- that the larger the total number of activists and the smaller the total number of loyalists in the system, the greater would have been the relative strength of the opposition.

Table 5

CALCULATED SIZE OF THE OPPOSITION WITH VARYING NUMBERS OF LOYALISTS AND ACTIVISTS IN THE SYSTEM

Loyalists		Activists		
In Both Groups (assumed)	In the Opponents (calculated)	In Both Groups (given)	In the Opponents (calculated)	In the Opponents, as a Fraction of Total Activists
1,250,000	85,000	60,000	6,000	0.10
900,000	74,000	60,000	8,000	0.13
450,000[a]	60,000[a]	60,000[a]	10,000[a]	0.17
350,000	42,000	60,000	11,000	0.18
250,000	32,000	60,000	11,500	0.19
150,000	21,000	60,000	12,500	0.21
(given)	(calculated)	(assumed)	(calculated)	
450,000[a]	50,000[a]	60,000[a]	10,000[a]	0.17
450,000	94,000	120,000	36,000	0.30
450,000	157,000	180,000	83,000	0.46
450,000	210,000	240,000	140,000	0.59
450,000	250,000	300,000	200,000	0.67
450,000	279,000	360,000	260,000	0.72

Note:

[a]This was the actual distribution in 1953 (see Table 2), from which the equilibrium coefficients governing the other cases were derived.

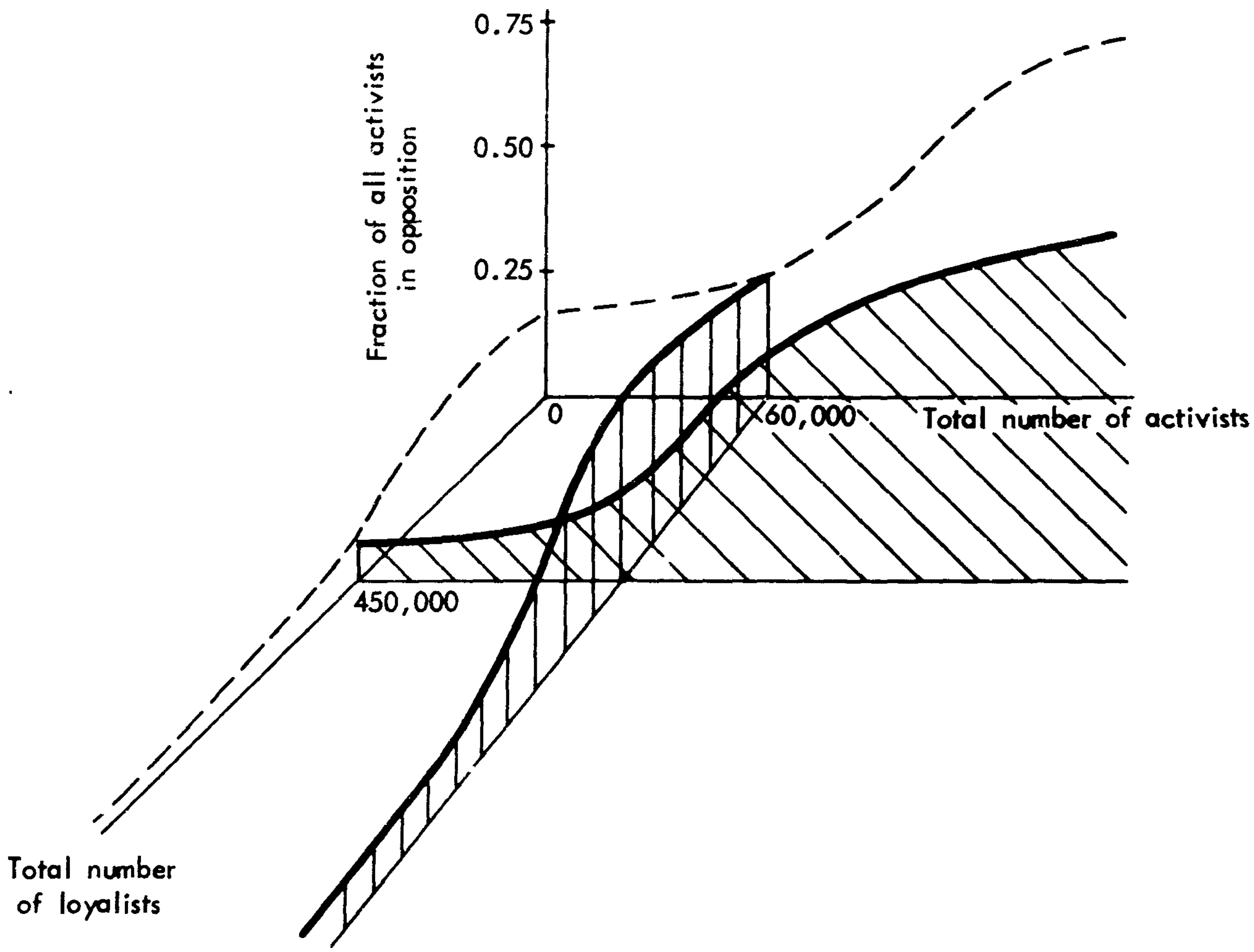

Fig. 1—Fraction of activists belonging to the opposition with varying numbers of politically conscious

IMPLICATIONS

Fig. 1 demonstrates that the tendencies of those who enter or leave the political arena are very important. If the great majority of entrants are of one temperament, their effect on the system may be just the reverse of what it would be if they are of another. The important distinction in our case was between the activity or passivity of the homo politicus: adding mainly activists yielded opposite results from adding mainly loyalists.* What personal tendencies are significant may vary from one society to another -- in one country the important distinction may be between those who are violent and those who are peaceful, in another between those who use one instrument and those who use another. But from this we see that we cannot say that any increase in political consciousness will be bound to increase the strength of one group at the expense of another. It depends upon the "nature" of the initial system, the stability of its parameters, and the relative importance of the different categories to the overall measure of political power. In the particular instance of Cuba the presence of individuals who tended to be loyal to a group but not active in its behalf favored Batista and the presence of those who tended to work actively favored the opposition, but the resultant favored the opposition to such an extent that it prevailed.

When the total number of migrating categories is more than two, any number of categories from one to the total-less-one can have an affinity for one particular group. In terms of the value of

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*Shapley and Shapiro have proved the theorem that if the proportion in which the different categories are added remains constant, trends in the ratios of the categories in the two groups will never reverse. For example, if adding a certain number of individuals in one set of proportions results in an increase in the relative number of members in group I, adding more individuals in the same set of proportions will result in a still further relative increase in group I's membership. So long as the proportions of entrants stays constant (and so long as the equilibrium coefficients remain unchanged too), group I will continue to gain indefinitely. This theorem holds even if the number of categories is multiplied -- provided the proportions do not change, neither will the trend.

the equilibrium coefficients, all but one can be less than unity, or all but one more than unity.* A group could never benefit from the addition to the system of individuals from any and all categories, but only up to a maximum of one less than the total number. Fated as a group may be, there is always one category which it has an affinity for. If this affinity is strong enough, by proselytizing this one type of individual it may offset the greater attractions of the other group for other types. Had Batista attracted enough sympathy for, or created enough fear of opposition to, his régime, he might have been able to have offset the effects of the entry of those whose nature led them to be active. Concessions to the majority of the people who were normally indifferent to their government might have compensated for increases in the number of those who were agitated and therefore tended to flock to the opposition.

We can also draw some implications concerning the permanence of the system, meaning by permanence not the absence of shifts in the balance of power but rather the continued existence of both groups under all stresses. The Cuban political system as defined in 1953 exhibits permanence, for with a preponderance of activists Batista's régime would still have been able to command some support while with a preponderance of loyalists the opposition would still have existed.** Finally, if among several groups there is one that is quite small, at least one of the groups will be less likely to be able to maintain itself than if all the groups were of roughly equal size.***

The relative numbers of the committed also affect the sensitivity of the system; i.e., the magnitude of its response to additions or subtractions of members. In general, the larger the proportion of

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*If there are movements from one category to another -- e.g., activists become passive or *vice versa* -- then this generalization does not hold.

**It was only when those committed to Batista fled or were liquidated that the system degenerated.

***Unfortunately one can say little about the effects of increasing the number of categories (or groups, or both) other than that the permanence and sensitivity of the system do not appear to be altered substantially. It is the tendencies of the individuals and the affinities of the groups that count, not their variety.

immobile individuals -- in chemical parlance the higher the concentration of species to which the barrier is impermeable or, what is the same, for which the Ks are zero or infinite -- the more likely is the system to be insensitive to changes. However, if there are some categories with very high or low (but not zero or infinite) Ks, the system will be altered greatly by their addition or subtraction. If the cost of moving from one group to another is very low (i.e., if the Ks are close to unity) for <u>all</u> the categories, then slight changes in the values of any of the equilibrium coefficients will produce substantial migrations.

STABILITY OF THE PARAMETERS

Just how slight need be changes in the values of the parameters to produce substantial shifts in political allegiance? This question must be asked, if not out of curiosity, then because the conclusions derived from numerical results in the Cuban case are dependent upon reasonable stability of the equilibrium coefficients.

Looking at the Jacobian matrix of partial derivatives with respect to the equilibrium coefficients $\frac{\partial N_I}{\partial K_A}$, $\frac{\partial A_I}{\partial K_A}$ ..., $\frac{\partial N_I}{\partial K_L}$ ..., etc. , we find that the distribution of membership in the two groups was fairly sensitive to changes in the equilibrium coefficients. If, for example, K_A had been increased in value by 0.01 (from 0.65 to 0.66), which would have been equivalent to increasing the relative attractiveness of Batista's régime to activists by 1.5 per cent, it would have prompted a shift of allegiance from opposition to support of the existing government of 1,010 individuals (i.e., $\frac{\partial N_I}{\partial K_A} = -1{,}010$; $\frac{\partial N_{II}}{\partial K_A} = 1{,}010$), 0.19 per cent of the total in the system. The effects of the migration would have been to reduce the membership of the opposition by 1.6 per cent and increase that of the existing régime, which was larger to begin with, by 0.21 per cent. As might be expected, changes in the equilibrium coefficients produce roughly proportionate shifts in allegiance.

How stable were the equilibrium coefficients in Cuba? Between 1953 and 1956 they apparently did not change at all: the distribution of members in 1956 calculated on the basis of the coefficients derived from 1953 was substantially correct. But what happened beyond 1956, when the political situation became even more fluid?

The subsequent date for which we tried to obtain an empirical estimate of the distribution of allegiance was two years later, June-July 1958, when the various forces in opposition to Batista agreed to unite, instituting the Pacto de Caracas. By that time the existing régime had lost about half of its membership of 1953, the loss being felt equally in each of the categories, while the opposition had slightly more than doubled its strength (see Table 6; the figures, like those of Table 2 and those corroborating Table 4, are subjective). If we calculate the equilibrium coefficients for June-July 1958, we find that $K_L = 1.15$ (versus 1.04 for 1953) and $K_A = 0.57$ (versus 0.65 for 1953). As can be inferred from the sensitivity of the system to changes in the values of the equilibrium coefficients measured above, the magnitude of the changes was not very great. But the direction of change is interesting: in the interval between 1956 and June-July 1958 the existing régime became even more attractive to those who were passive by nature (perhaps because it represented stability and order?) and even more antithetic to those who were immoderate. Consequently, as combat increased, each group tended to attract in ever greater proportions one sole type of adherent, the groups becoming polarized temperamentally as well as ideologically in the process.

THE DEGREE OF STRESS IN THE SOCIETY

At the beginning of this paper, we stated that one of our aims would be to account for the fact that the system might behave differently in periods of acute stress. We shall not be able to allow for changes in the state of tension explicitly because to do so would increase the complexity of the model to the point where it could not be solved analytically; but we can at least indicate in terms of the nature of the system itself how changes in the political atmosphere might be expressed. Until now we have assumed that individuals were

Table 6

OBSERVED DISTRIBUTION OF FOLLOWERS AND
OPPONENTS OF THE CUBAN RÉGIME,
JUNE-JULY 1958

Category of Members	Opponents	Followers of Batista's Régime	Total
Loyalists	100,000	200,000	300,000
Activists	25,000	25,000	50,000
Committee	10,000	10,000	20,000
Total	135,000	235,000	370,000

[redacted]

fixed in their tendencies -- that they were either loyal or active or committed irrevocably. We permitted activists to join either political group, but we did not permit them to change to loyalists or committed. Yet this sort of a shift is customary during a period of great stress, for under tension some individuals who have been passive become active (and to a certain extent the reverse as well), and some who have been immobile become mobile.

If we wished to construct a chemical model analagous to a political system under great stress, we should have to include movements from one category to another: i.e., intra-group as well as inter-group reactions. An example would be raising the temperature of the system to the point where molecules would not only pass and repass through a barrier between two phases but would also begin to react one with another. At the elevated temperature the equilibrium coefficients covering the chemical reaction between the molecules would take on significant, rather than zero or infinite values.

Increasing the freedom of movement necessitates obtaining more information. The affinities which we needed to know in the Cuban case were only those of the category of individual for the two groups. If we were to acknowledge movement between categories as well, we should also have to concern ourselves with the affinities of individuals for different types of political action. Yet to describe adequately what occurred in Cuba towards the end of 1958 we might have to do just this, because many of the members of the armed forces and the <u>guardia rural</u> who had worked actively for Batista began to adopt a passive role or even disassociated themselves from his régime. Many students who had formerly been merely sympathetic to the opposition became active on its behalf. It was probably these changes in the type of political activity, as well as in the total membership of the groups, that increased the swing towards the opposition and made more conclusive its victory.